Work, Energy, and Power

Agenda

Energy conservation, conversion, and transformation

- Potential energy, kinetic energy, work, and power
  - Variable force, chemical rearrangement energy (Enthalpy)
  - Examples

- Kinetic energy transfer,
  - Dissipation, randomization and spontaneous processes
  - Examples of thermal motion, Maxwell-Boltzmann distribution

- Electricity and Electromagnetic Power
  - Electric fields and currents, metallic and semiconductors
  - Magnetic induction
  - AC circuits

- Thermodynamics principles and applications
  - First Law & Second Law of Thermodynamics, Entropy
  - Transfer of thermal energy (heat)
  - Conduction, convection, radiation (cooling)
  - Internal energy, equivalence of work and heat

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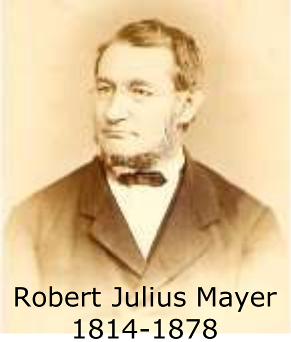
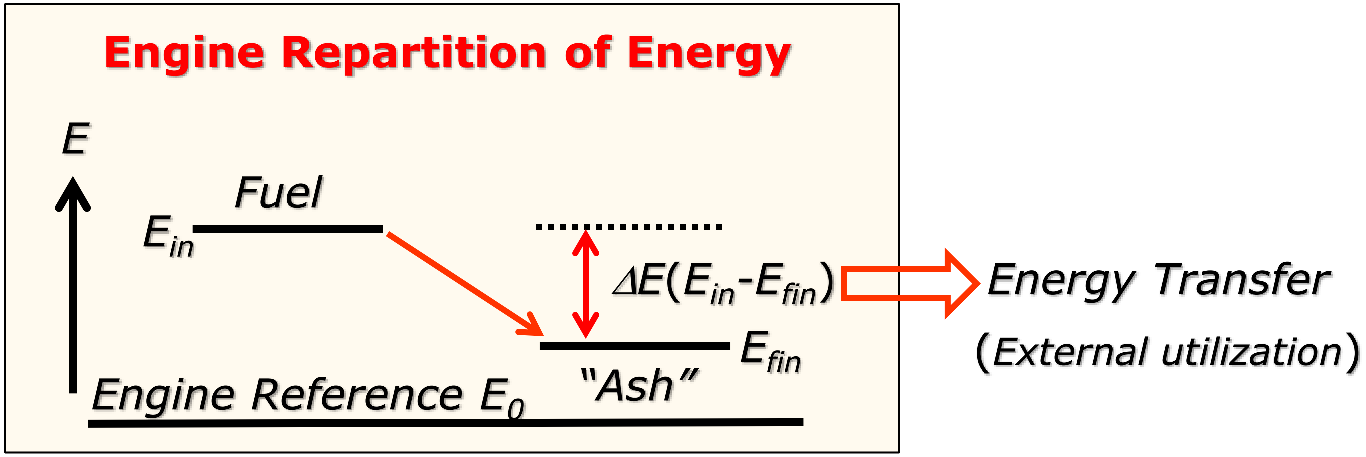
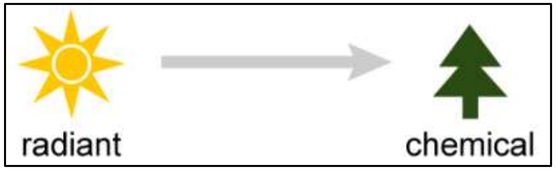
Schematic of Human Energy Utilization

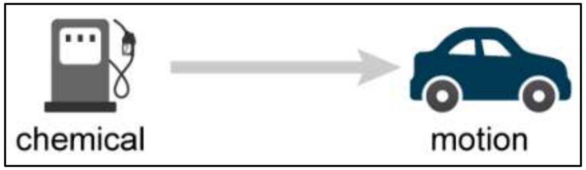

Robert Julius Mayer
1814-1878

Conservation of Energy ≙ 1st Law of Thermodynamics (TD)
Energy in an isolated system can never be created or destroyed.
→ It can only be transformed.

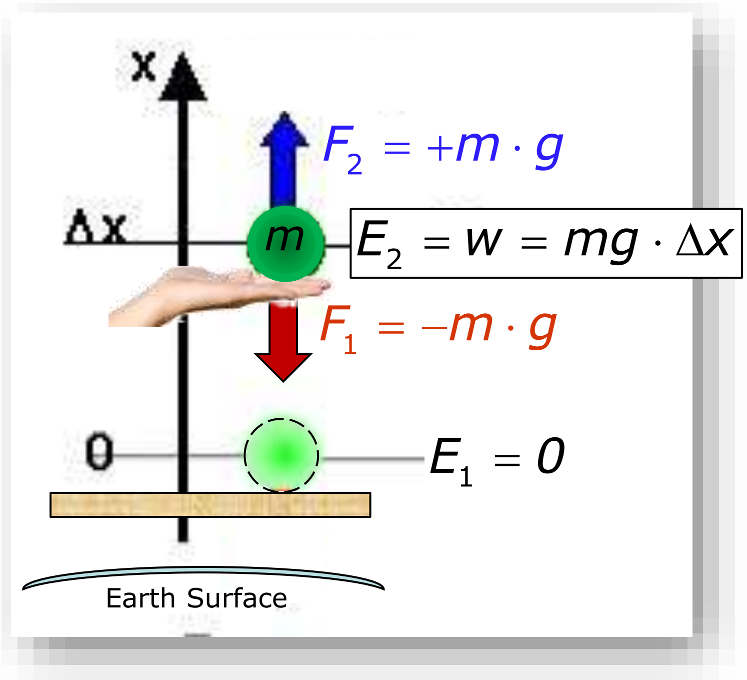

Engine Repartition of Energy

Examples
For Energy
Conversion





Mechanical Work & Energy: Weightlifting



Only differences Δ*E* in energy *E are* measureable →
→ arbitrary energy-zero, *E=0*.

Newton's Law→ Motion of massive bodies:

$$\boxed{Force = Mass \cdot Acceleration}$$

Balancing gravitational force F_1 requires an equal force F_2 in the opposite direction.

$$\vec{F}_2 = -\vec{F}_1 = -m \cdot g$$

Applying $\boldsymbol{F}_2$ over an altitude change **Δx** requires work ***w***

$$w = +F_2 \cdot \Delta x = -F_1 \cdot \Delta x = m \cdot g \cdot \Delta x$$

This increases the intrinsic (potential) energy of the body (= system) by

$$\boxed{\Delta E = m \cdot g \cdot \Delta x \rightarrow w \equiv \Delta E}$$

If the work ***w*** is done during time Δ*t*, the mean power applied is

$$\boxed{\Delta P = \Delta E / \Delta t = m \cdot g \cdot (\Delta x / \Delta t)}$$

The body gains, as internal energy, the difference in gravitational potential energies at different heights x. Body can do work *w=-mgΔx*.

Numerical Example

Q: What is the potential energy (*SI* units) gained by a body of mass *m = 1 kg* lifted vertically up from a table by a distance *h = 1m* ?

A: The force resisting the motion of the body is the gravitational force (downwards is the ***negative h direction***)

$$F_g = -m \cdot g = -1\,kg \cdot 9.81\,m/s^2 = -9.81\ N$$

unit

To lift the body requires a ***force F upwards*** of the magnitude *9.81 N*.

This force ***F = - F_g =+9.81 N***, applied over the distance of ***1m*** does the work

$$w = F \cdot 1m = 9.81\,Nm = 9.81\ J = 6.12 \cdot 10^{19} eV > 0$$

Q: If this work is done within *1* second, what is the power *P* exerted?

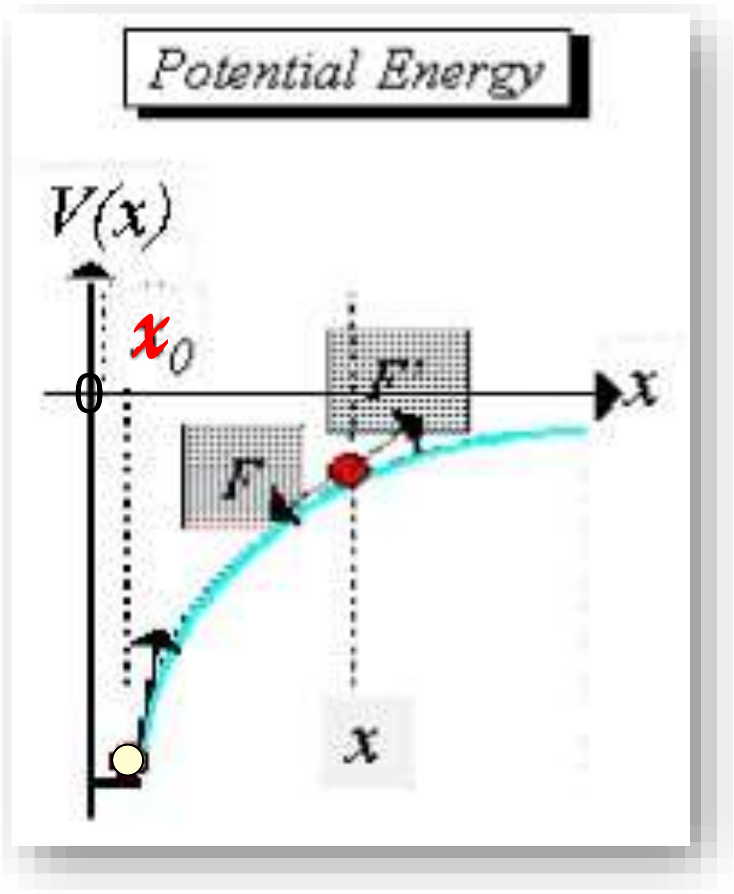
A:

$$P = w/1s = 9.81\ J/s = 9.81W\ (Watt)$$

Q: If the body is then dropped from its *1-m* height, what is its velocity hitting the table (h=0)?

$$w = 9.81J = 0.5 \cdot m \cdot \upsilon^2 = 0.5kg \cdot \upsilon^2 \rightarrow \upsilon = \sqrt{9.81Nm/0.5kg} = 4.4\,m/s$$

Work Against a Variable Force



Variable force F′(x), differential work:
Sum over (infinitely) many differentials *dw*

Lifting: $dw = F'(x) \cdot dx = -F(x) \cdot dx > 0$

→ Total work ***done on*** particle
in terms of potential energy difference:

$$\mathbf{w} = \int_{x_0}^{x} dw(x') = \int_{x_0}^{x} F'(x') \cdot dx' = -\int_{x_0}^{x} F(x') \cdot dx' \mathbf{>0}$$

$$w = -\int_{x_0}^{x} F(x') \cdot dx' = \int_{x_0}^{x} \frac{dV(x')}{dx'} \cdot dx' = V(x) - V(x_0) = \Delta V$$

$$\textit{In 3D components}: \; F \to \vec{F} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \; ; \; x \to \vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$w = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{\vec{r}_0}^{\vec{r}} \vec{\nabla} V(\vec{r}) \cdot d\vec{r} = V(\vec{r}) - V(\vec{r}_0)$$

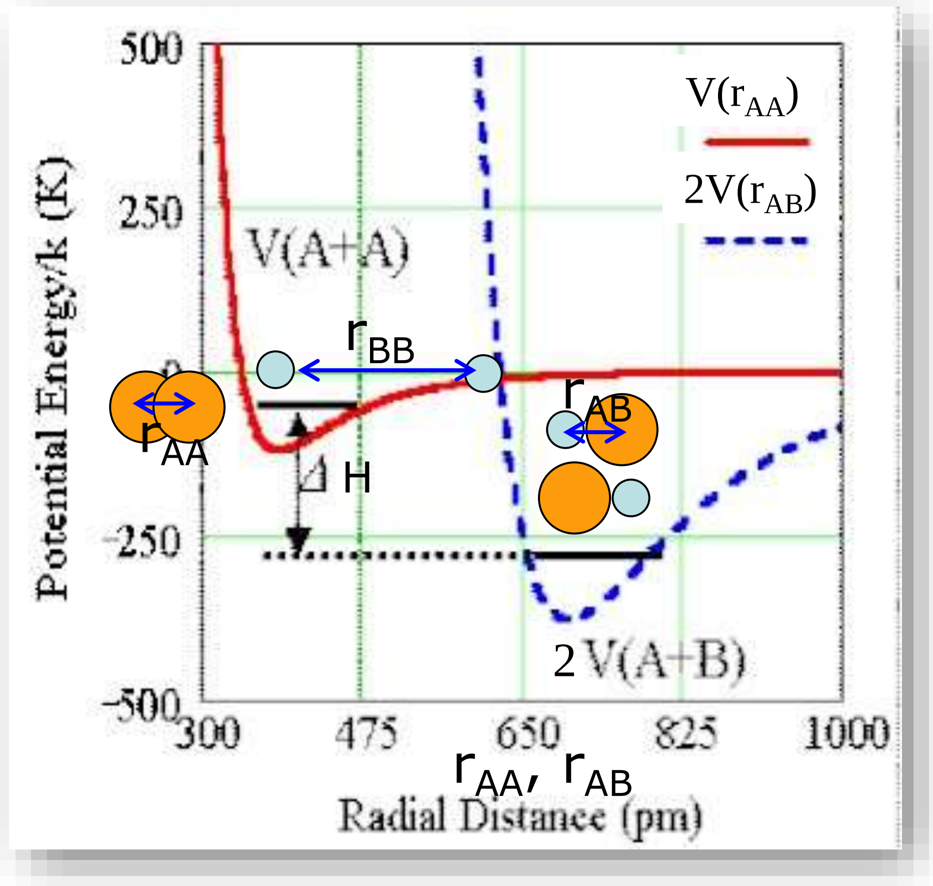
Force =
Negative gradient:

$$F(x) = -\frac{dV(x)}{dx} \to 3D: \quad \vec{F}(\vec{r}) = -\vec{\nabla} V(\vec{r}) = -\begin{pmatrix} dV(\vec{r})/dx \\ dV(\vec{r})/dy \\ dV(\vec{r})/dz \end{pmatrix}$$

Del or ***Nabla* Operator**

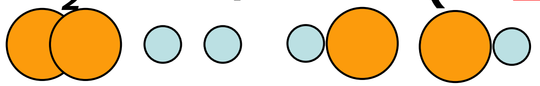
Energy Gain in Chemical Configuration Changes

Example: Covalent bonding
(Lennard-Jones potential)



Consider schematic reaction between one bound molecule $\boldsymbol{A_2}$ and 2 unbound atoms ***B*** forming 2 bound molecules ***AB***:

$$\boldsymbol{A_2} + 2B \rightarrow 2\boldsymbol{AB} \ \ (+\Delta H)$$



Enthalpy ΔH < 0: energy released from the molecular system (2***AB***)

Since *B* are individual atoms and not bound together, $V(B)=V_{B+B}=0$.

Reaction takes place if

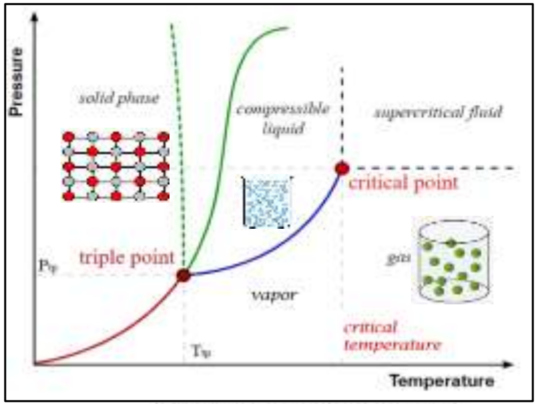
$$2V_{A+B} < V_{A+A} + \underbrace{V_{BB}}_{=0} = V_{A2}$$

Reaction is exo-thermic (final system is more strongly bound).

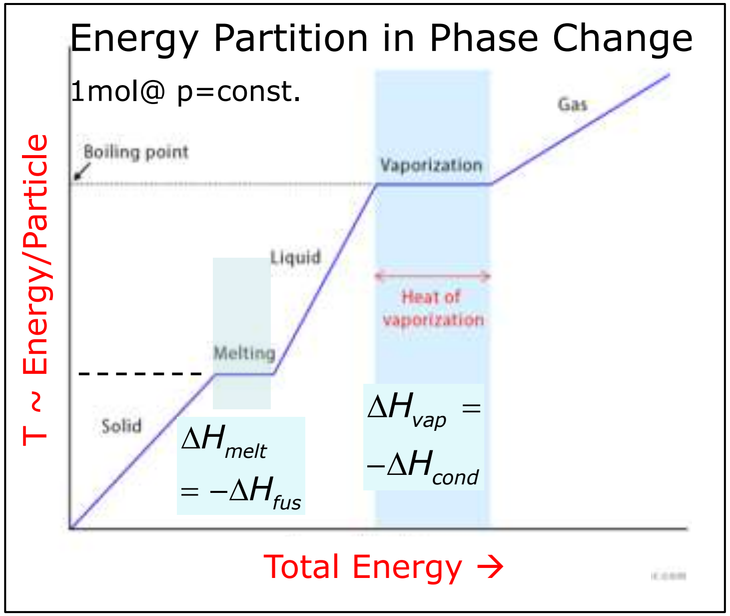
Rearrangement of individual constituents of ensemble of atoms and molecules is associated with changes in interaction potential (bonding) energy.

Energy Repartition in Physical Configuration (Phase) Changes

Phase Diagram H_2O



		Critical pressure P_c		Critical temperature T_c		
Material	Formula	psia	bar (abs)	°F	°C	$k = C_p/C_v$
Water	H_2O	3206	221	705	374	1.32



Energy Partition in Phase Change

1mol@ p=const.

$\Delta H_{melt} = -\Delta H_{fus}$

$\Delta H_{vap} = -\Delta H_{cond}$

$$Internal\ structural\ energy: 1mole\ substance$$

$$extensive\ (additive)\ state\ function\ energy\ U$$

$$Enthalpy = Structural\ Energy\ +$$
$$+\,p-V\,work\ to\ access\ space$$
$$\rightarrow \quad H = U + P \cdot V$$

$$Process\ @\ p = const \rightarrow \Delta H\big|_p = \Delta U + p \cdot dV$$

$$H = \text{"}extensive\text{"}\ state\ function$$

$$Reference\ \text{"}zero\text{"}\ energy\ \ H^0\ @\ standard\ state$$
$$T = 25^0C,\ P = 1bar \leftarrow large\ databases$$
$$H^0 = 0\ for\ pure\ elemental\ substances$$

$$Phase\ changes \rightarrow \Delta H \neq 0$$

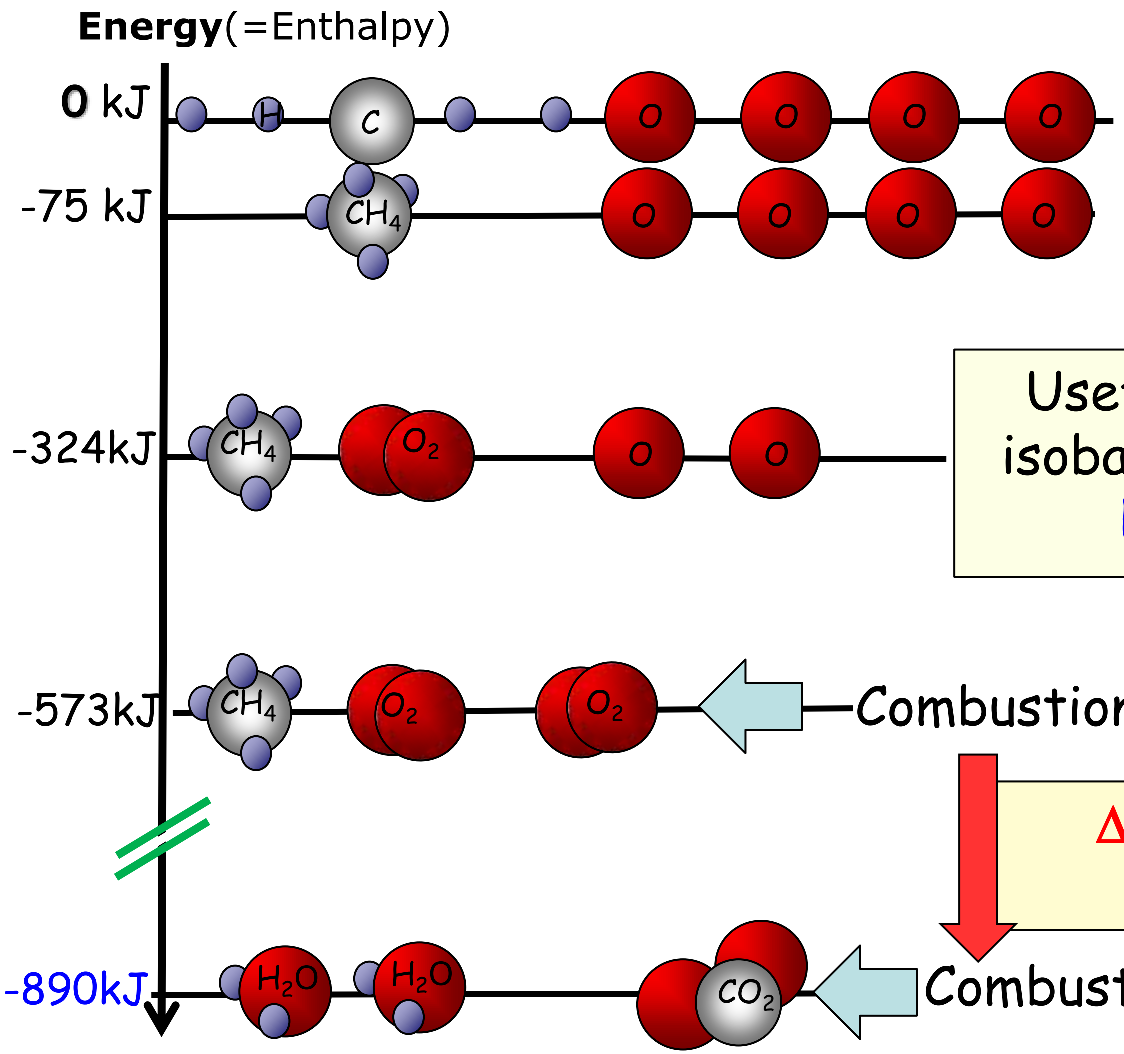
$$\boxed{latent\ heat\ transfer} \rightarrow$$
$$\Delta U\ to\ change\ internal\ structure$$
$$@\ phase\ transition: \Delta T = 0$$

$$@\ 1\,bar,\ T = 100^0C \quad (\neq the\ standard\ state)$$
$$H_2O(\ell) \rightarrow H_2O(g) \rightarrow \Delta H_{vap} = +40.7\,kJ/mol$$
$$H_2O(g) \rightarrow H_2O(\ell) \rightarrow \Delta H_{cond} = -40.7\,kJ/mol$$

Example: Bond Changes in Combustion

Example: burning natural gas $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O$ *(not spontaneous)*

Methane



Constituents in unbound (free) states =Reference

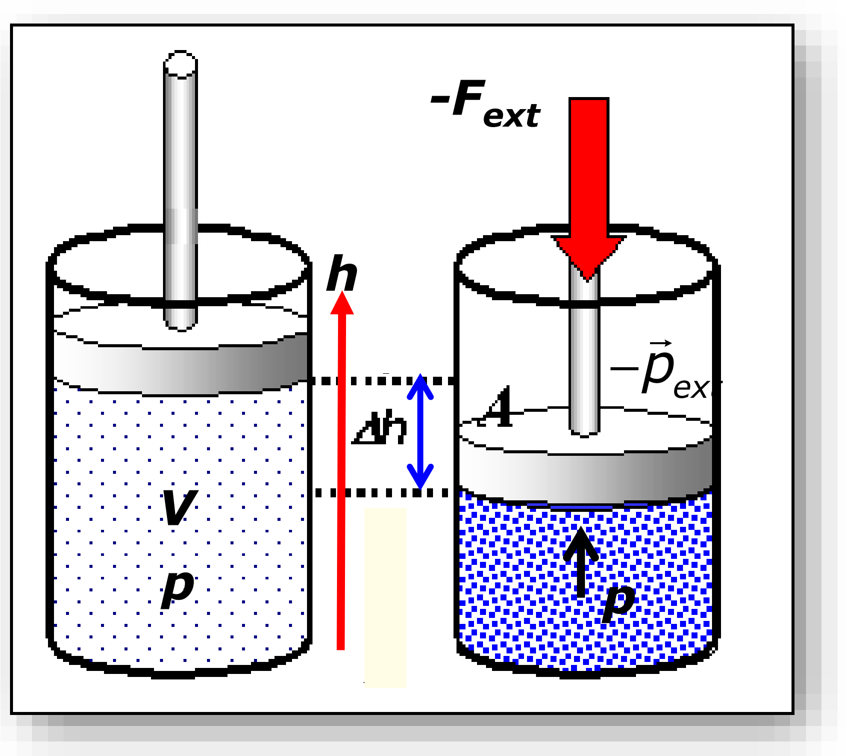
Useful energy function for isobaric gas phase reactions:
Enthalpy H = U+p·V

Combustion Initial State Bond Energy

$\Delta H = q < 0$ **Energy Release**

Combustion Final State Bond Energy

Kinetic Energy Changes in Compression



Gas=equilibrated ***system of independent particles moving in random directions.***

Compression of a gas volume *V* with a constant force *F* (e.g., weight) on a constant area *A*:

→ Pressure *p* = Force *F*/Area *A*,

at *p* = *p*$_{ext}$ =const.
(external, not internal)

$$p = \frac{-F_{ext}}{A} = p_{ext} \rightarrow \Delta V = A \cdot \underbrace{\Delta h}_{<0} < 0$$

Compression work done on system

$$w = -F_{ext} \cdot \Delta h = -\left(p_{ext} \cdot A\right) \cdot \Delta h = -p_{ext} \cdot \underbrace{\Delta V}_{<0} > 0$$

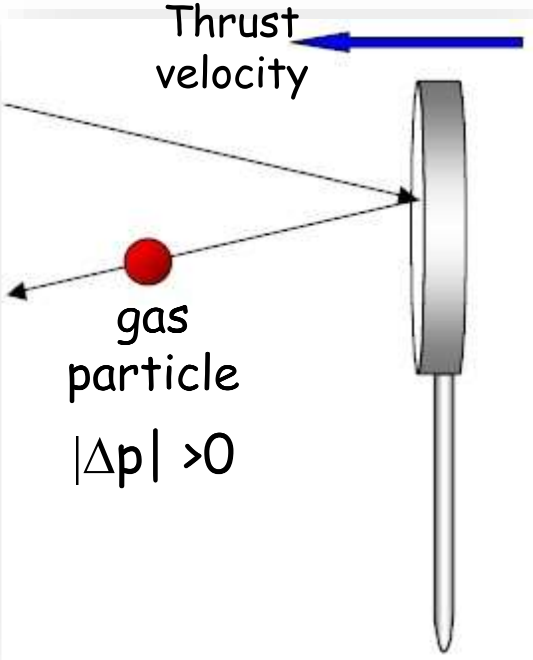
Sign Convention: Compressional work on a gas volume (=system) increases the internal energy ***E*** of the gas .

Therefore, work *w > 0* is counted as positive (done on gas).

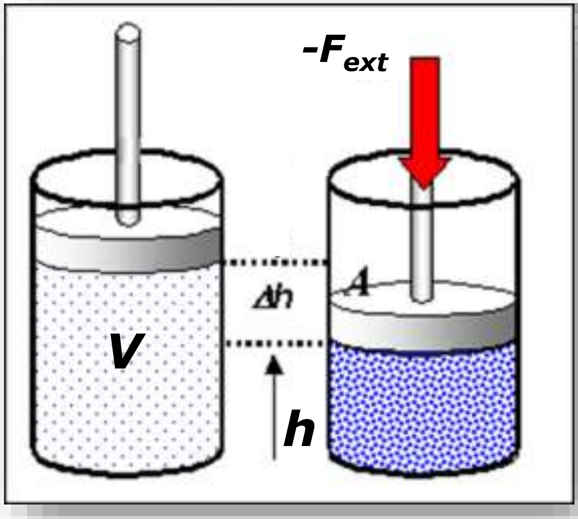
Q: How can we measure the gas pressure?
Q: What happens to the piston, what is the pressure *p* inside the gas volume?

Compression Work as Energy Transfer

Energy and momentum are transferred to gas particles hit by a (collectively) moving piston, depending on the relative velocity of piston and gas particle.



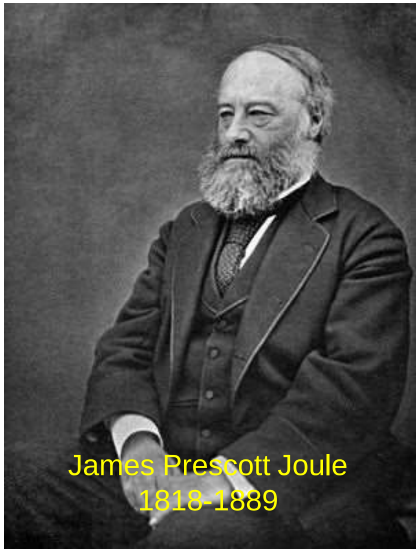
The effect is similar to that driving a tennis ball with a racket.



Thermally insolated system (cylinder with gas)

Compressed gas has more energy in random motion than before. Transfer collective → random motion = dissipation of collective energy into heat.

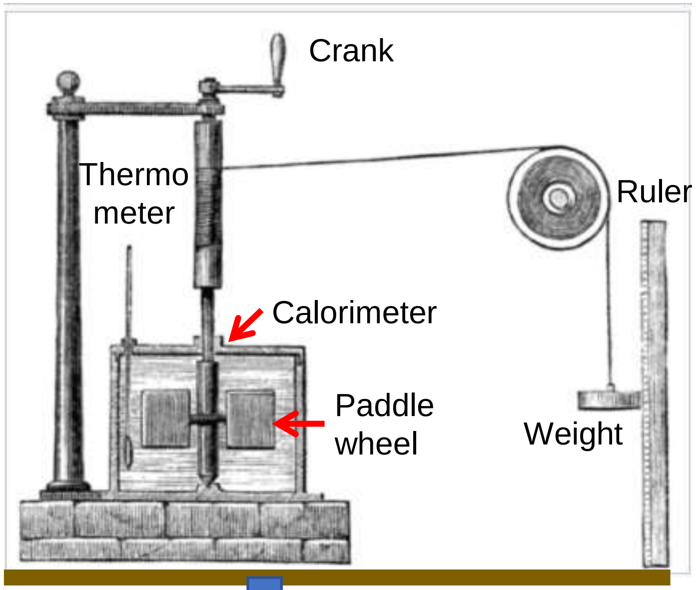
Mechanical Equivalent of Heat



"An Experimental Enquiry Concerning the Source of the Heat which is Excited by Friction", (1798), Philosophical Transactions of the Royal Society p. 102

$$\text{Work } \ W \propto Q \ \text{ Heat}$$
$$\Rightarrow W = JQ$$
$$\Rightarrow J = \frac{W}{Q}$$

Joule's experiment



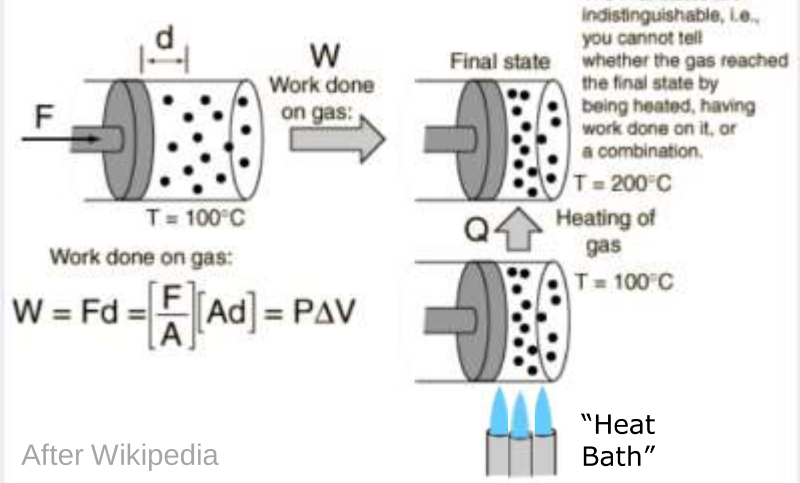
$$J = 4.186 \ \ kJ/kcal$$

Specific heat → Energy(ΔT): $q = C \cdot \Delta \boldsymbol{T}$

Heat energy $\boldsymbol{q}$ required for heating 1g material by $\Delta T = 1^0 C$

of H_2O: $C_V(\ldots) \approx (4.17\text{-}\ 4.22)\ J/(g \cdot {}^\circ C)$

P-V Work



The final states are indistinguishable, i.e., you cannot tell whether the gas reached the final state by being heated, having work done on it, or a combination.

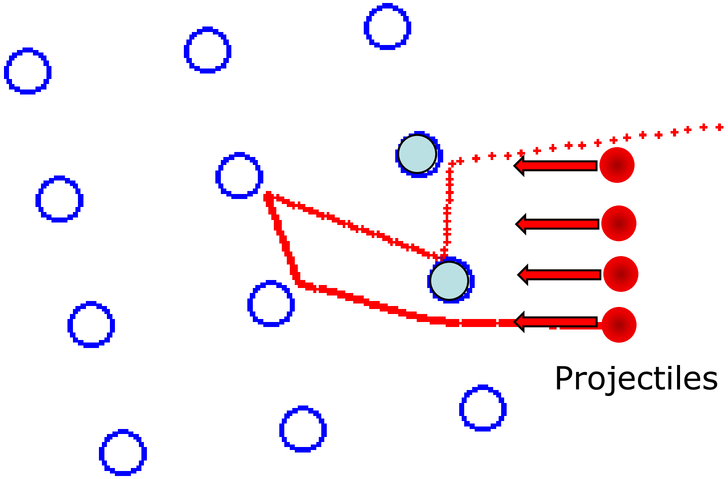
Work done on gas:

$$W = Fd = \left[\frac{F}{A}\right]\left[Ad\right] = P\Delta V$$

After Wikipedia

Energy Dissipation (E-Randomization, 2nd LTD)

Multiple Scattering of Projectile @ Fixed Lattice Atoms



A lattice-like ensemble of bound atoms or ions is hit by fast projectiles (or photons).

Sim: Several “projectile” particles enter the system at various initial conditions: positions from the right with identical momenta (e.g., kicked by a racket).

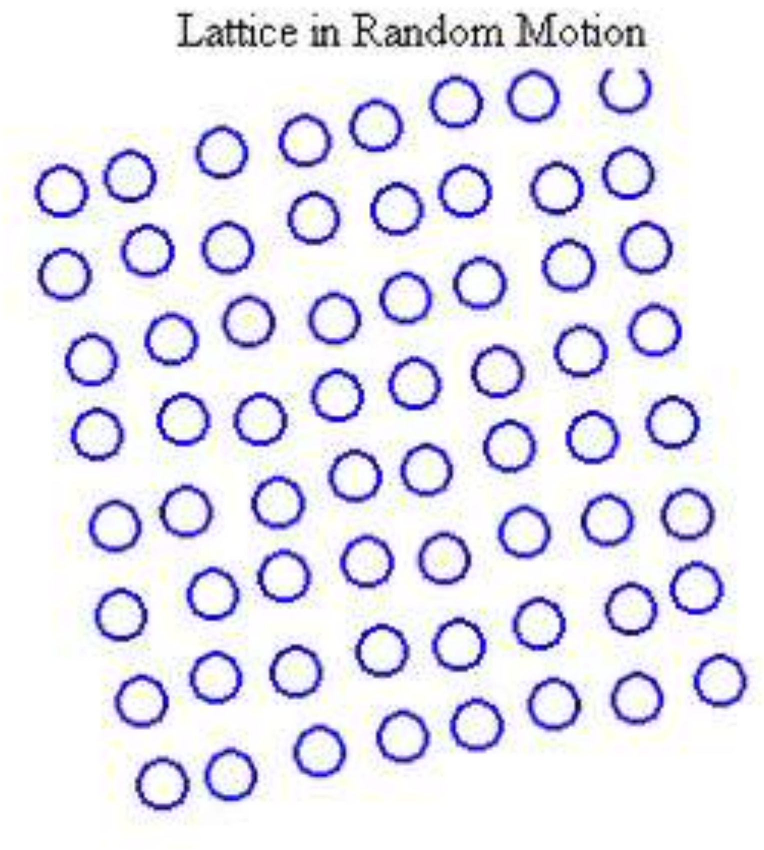
Depending on how and where the first lattice particle is hit, the next few collisions and their momentum and energy transfers change.

Lattice_Scattering.avi

Multiple scattering → random pattern of projectile deflections, momenta and energies. → Ensemble collision partners receive recoil momenta, are accelerated and deflected. They now behave like projectiles and collide with other ensemble particles. An avalanche of secondary collisions dissipate the initial projectile energy over all particles → system becomes “heated”.

$$\mathit{Temperature} \triangleq \left\langle \mathit{kinetic\ energy} / \mathit{particle} \right\rangle$$

Thermal Motion in Solids



Hot lattice @ thermal equilibrium: individual constituents (atoms or ions) have the same "thermal energy spectrum."

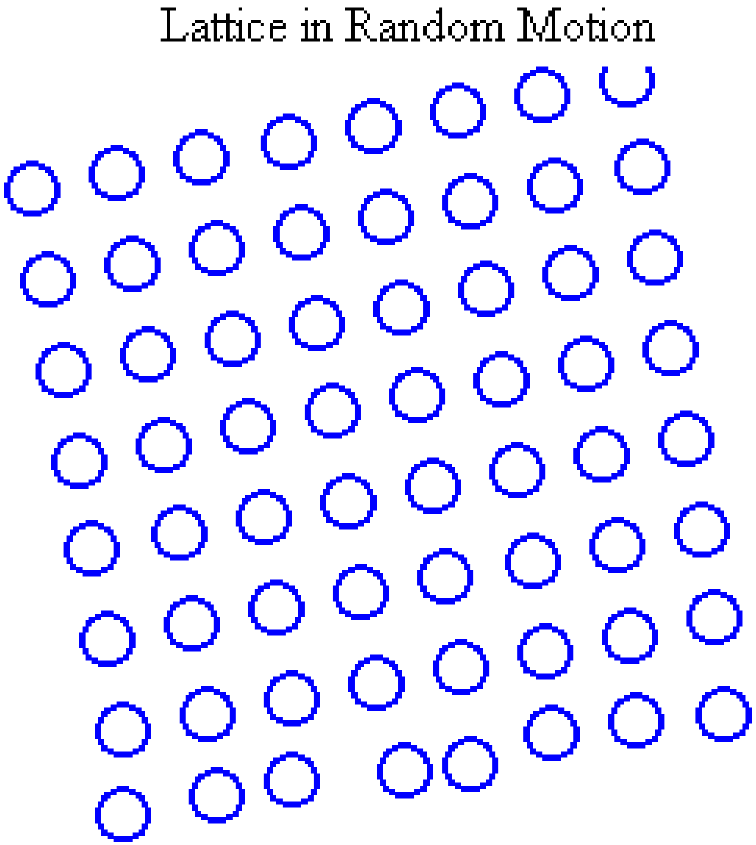
Energy injected from the outside surroundings dissipates and increases displacement motions of lattice particles about their equilibrium sites.
→ Average kinetic (motional) energies
→ increases temperature ***T***.

At higher excitations *T*, the structure begins to disappear, the lattice is "melting", becomes a liquid and, eventually, a gas (vapor).

The average kinetic energy of each particle is known as "temperature *T*" (units of k_B)

$$\boxed{\textit{Temperature} \hat{=} \left\langle \textit{kinetic energy} / \textit{particle} \right\rangle}$$

Thermal Motion in Solids



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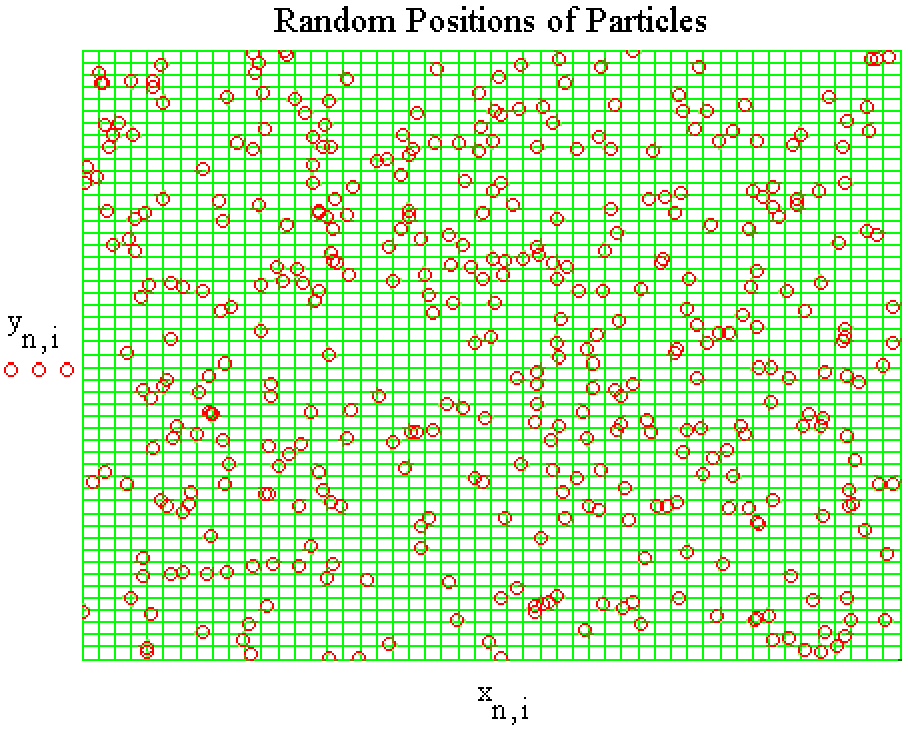
$$\textit{Temperature} \triangleq \left\langle \textit{kinetic energy}/\textit{particle} \right\rangle$$

Random (Thermal) Motion in Space



Rnd_Positions.avi

Example: Motion in two dimensions of 300 non-interacting (ideal-gas) particles.

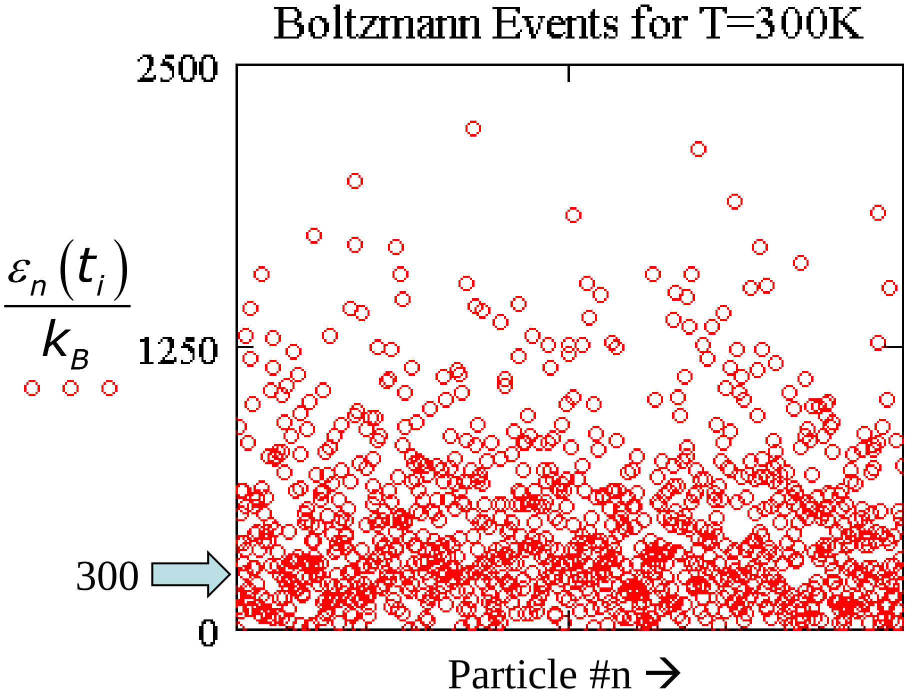
All particles move in random directions because of multiple collisions, which actually do occur but are not explicitly treated here.

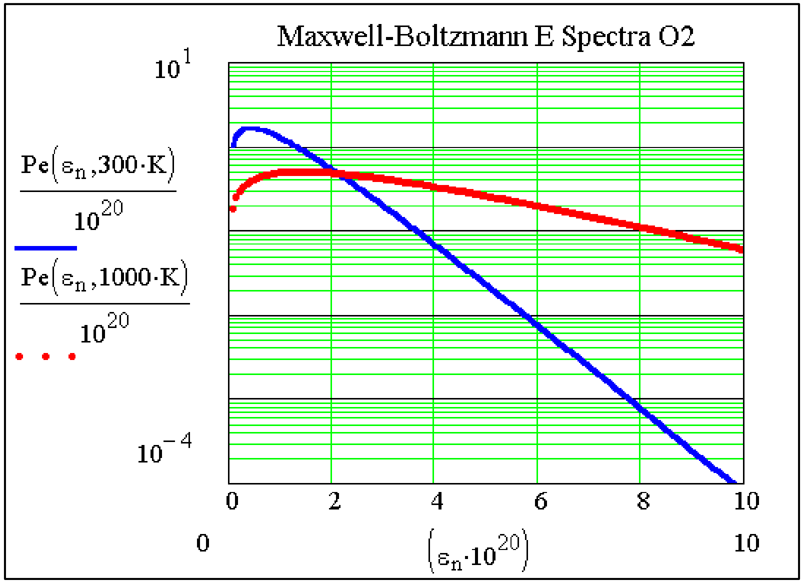
Every particle visits every one of the energetically equivalent cells.

Contrast: Collective motion.

Particles in a gas move in different directions and at different speeds, colliding with one another often. Eventually, their positions at any given time are random. All available (accessible) space is visited by all particles, in due time.→ Ergodic Theorem

Gases in Randomized State (Thermal Equilibrium)



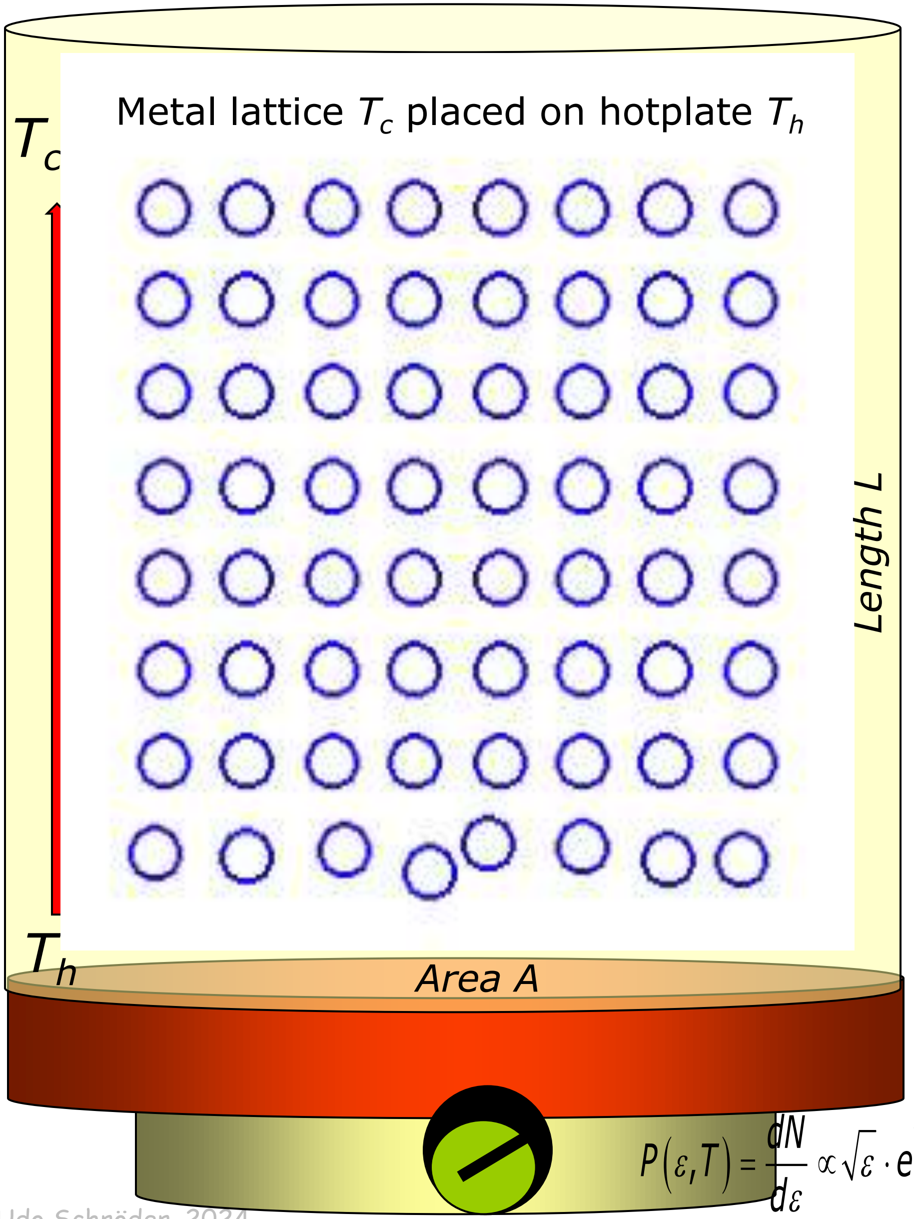


$$P(\varepsilon,T)=\frac{dN}{d\varepsilon}\propto\sqrt{\varepsilon}\cdot e^{-\frac{\varepsilon}{k_B T}}$$

Particles in a gas collide continuously, all kinematically possible momentum and energy transfers occur. → randomly fluctuating (“thermal”) kinetic-energy spectrum → Universal “Maxwell-Boltzmann” distribution.

(Real gases with internal structure have internal energy states populated with a weight $\exp\{-\varepsilon_n(t_i)/k_B\cdot T\}$ Boltzmann factor

Thermal Energy Conduction in Solids



$$P(\varepsilon,T)=\frac{dN}{d\varepsilon}\propto\sqrt{\varepsilon}\cdot e^{-\frac{\varepsilon}{k_BT}}$$

Materials touching hot object absorb (or dump) heat energy via propagating density disturbances → equilibrate

Energy *q* flow: hot → cold

$$\frac{dq}{dt}=-\kappa A\frac{(T_c-T_h)}{L}\rightarrow conductivity\ \kappa$$

$$3D:\quad \vec{j}_q=\frac{d\vec{q}}{A\cdot dt}=-\kappa\cdot\vec{\nabla}T$$

Material expands in spatial dimensions

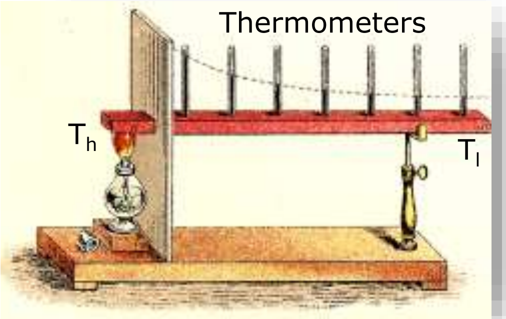
$$V(T)=V(0)\cdot(1+\alpha\cdot T)$$

$$\alpha=volume\ expansion\ coeff$$

Systems at any temperature emit thermal radiation (photons), which can be absorbed (possibly re-emitted) by objects in the environment.

Remember: Stefan-Boltzmann Law

Transfer of Randomized Energy: Heat Flow



Heat conduction, flux=current density through area *A*

$$Newton's\ Law\ of\ cooling\ \frac{dQ}{dt} = -h \cdot A \cdot (T - T_{ambient})$$

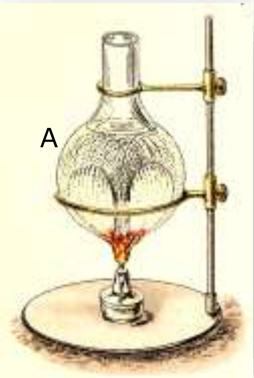
$$Heat\ transfer\ coefficient\ h\ (W/m^2K);\ area\ A$$

coordinate unit vectors

$$Fourier's\ Law:\ \vec{j}_q = -\kappa \cdot \vec{\nabla} T(\vec{r}) = -\kappa \cdot \left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k} \right)$$

$$Thermal\ conductivity\ \kappa\ (W/mK)$$

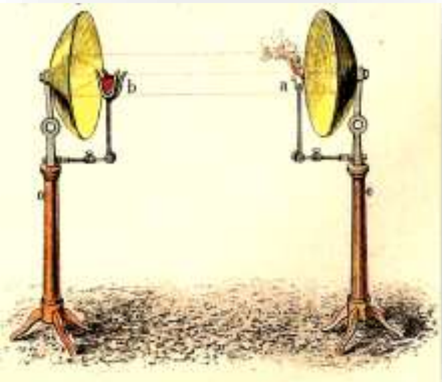
https://www.schoolmykids.com/learn/interactive-periodic-table/thermal-conductivity-of-all-the-elements



Heat convection: Heat transfer via particles (mass flow)

$$Newton's\ Law\ of\ cooling\ \frac{dQ}{dt} = -h \cdot A \cdot (T - T_{ambient})$$

$$Heat\ transfer\ coefficient\ h\ (W/m^2K);\ area\ A$$



Heat radiation: Heat transfer via photons

$$Stefan-Boltzmann\ Law$$

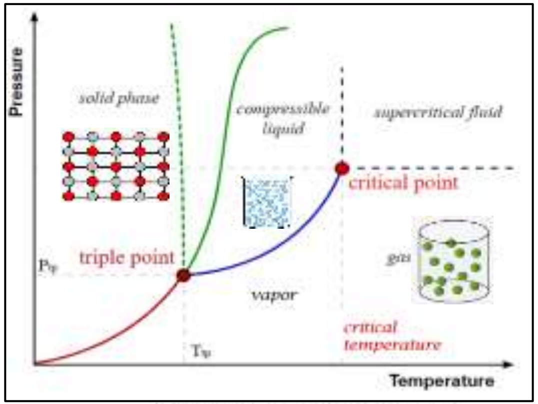
$$Radiated\ thermal\ flux\ j_Q = \varepsilon \cdot \sigma_{SB} \cdot \left(T^4 - T^4_{ambient}\right)$$

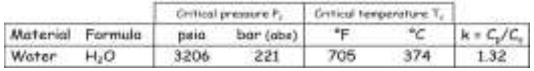
$$Emissivity\ \varepsilon\ (often = 1)$$

$$Stefan-Boltzmann\ constant\ \sigma_{SB} = 5.6703 \cdot 10^{-8}\, W/m^2K^4$$

Energy Repartition in Physical Configuration (Phase) Changes

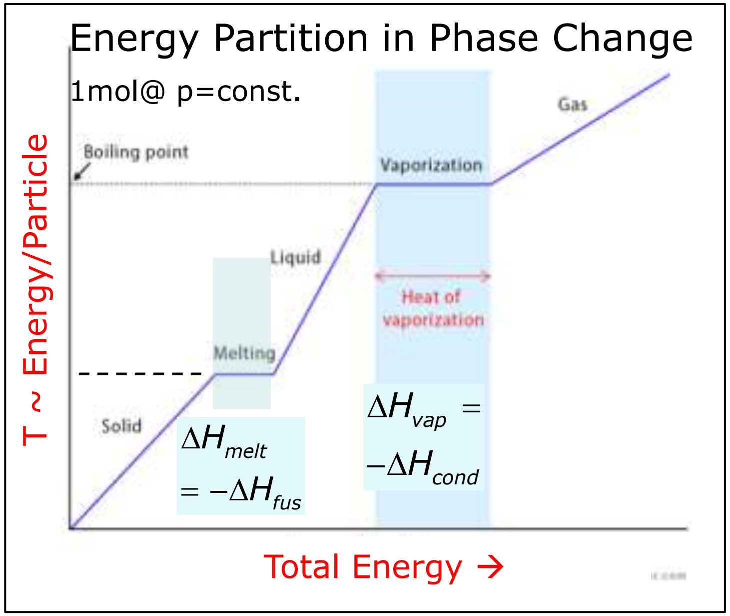
Phase Diagram H_2O





		Critical pressure P_c		Critical temperature T_c		
Material	Formula	psia	bar (abs)	°F	°C	$k = C_p/C_v$
Water	H_2O	3206	221	705	374	1.32

Energy Partition in Phase Change



$Internal\ structural\ energy: 1mole\ substance$

$extensive\ (additive)\ state\ function\ energy\ U$

$$\boxed{\begin{aligned} &Enthalpy = Structural\ Energy\ + \\ &\qquad + p-V\, work\ to\ access\ space \\ &\qquad \rightarrow \quad H = U + P\cdot V \end{aligned}}$$

$$Process\ @\, p = const \rightarrow \Delta H\big|_p = \Delta U + p\cdot dV$$

$$H = \text{"}extensive\text{"}\ state\ function$$

$Reference\ \text{"}zero\text{"}\ energy\ \ H^0\ @\ standard\ state$

$T = 25^0 C,\ P = 1bar \leftarrow large\ databases$

$H^0 = 0\ for\ pure\ elemental\ substances$

$Phase\ changes \rightarrow \Delta H \neq 0$

$\boxed{latent\ heat\ transfer} \rightarrow$

$\Delta U\ to\ change\ internal\ structure$

$@\ phase\ transition: \Delta T = 0$

$@\, 1\, bar,\ T = 100^0 C \quad (\neq the\ standard\ state)$

$H_2O(\ell) \rightarrow H_2O(g) \rightarrow \Delta H_{vap} = +40.7\, kJ/mol$

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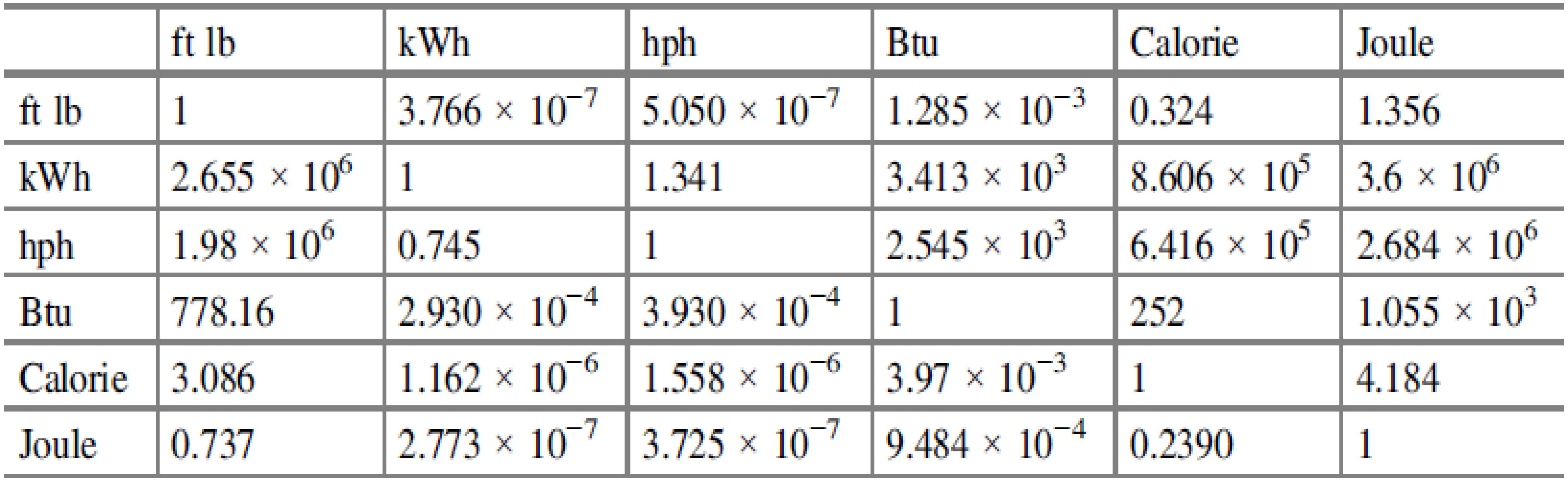

	ft lb	kWh	hph	Btu	Calorie	Joule
ft lb	1	3.766×10^{-7}	5.050×10^{-7}	1.285×10^{-3}	0.324	1.356
kWh	2.655×10^{6}	1	1.341	3.413×10^{3}	8.606×10^{5}	3.6×10^{6}
hph	1.98×10^{6}	0.745	1	2.545×10^{3}	6.416×10^{5}	2.684×10^{6}
Btu	778.16	2.930×10^{-4}	3.930×10^{-4}	1	252	1.055×10^{3}
Calorie	3.086	1.162×10^{-6}	1.558×10^{-6}	3.97×10^{-3}	1	4.184
Joule	0.737	2.773×10^{-7}	3.725×10^{-7}	9.484×10^{-4}	0.2390	1

End
Work/Energy/Power I