Agenda

Class time for regular lecture?

Take Radiation-Safety class/exam

- Lecture 2a: *Nuclear Radioactivity*
- Lecture 3: *Introduction to the data analysis software package Igor*
- Exercises with Igor (Jordan Butt)

- *Electronic signal processing, scintillation detectors*
- *Lect 2b*(cont'd), *Exercises Radioactivity*
- Igor exercises with γ spectra

Time Dependent Radio-Activity

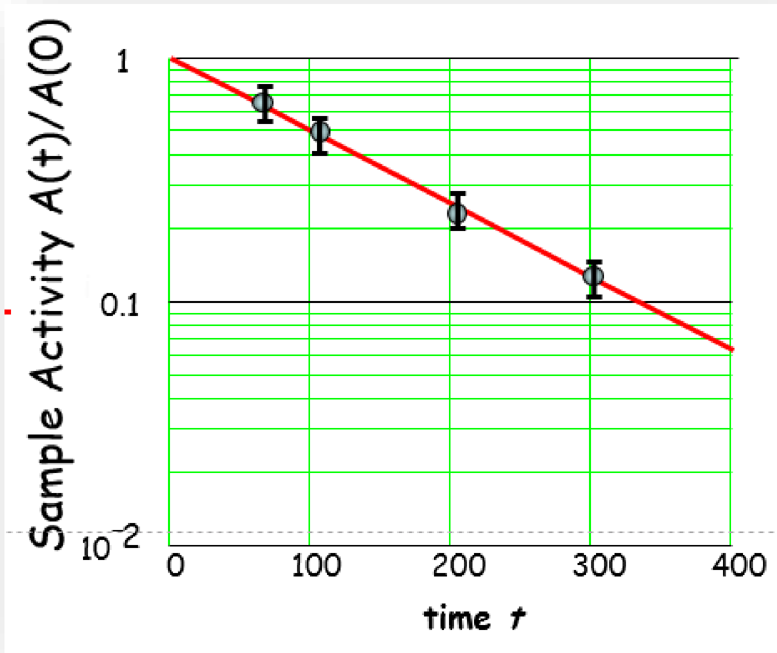
Mean Activity A of sample with N members $(ex: N = 10^{32}\ U - 238\ atoms)$

$\langle \#of\ decays \rangle / unit\ time$: *Mean lifetime* $\tau = \lambda^{-1}$

$$A(t) = \frac{N(t)}{\tau} \rightarrow N\ decreases\ with\ time$$

$$A(t) = -\langle \dot{N} \rangle = -\left\langle \frac{d}{dt} N(t) \right\rangle = \lambda \cdot N(t)$$

$$\langle N(t) \rangle = N(t=0) \cdot e^{-\lambda \cdot t}$$



Halflife $t_{1/2} = 0.6931 \cdot \tau$

$$\langle N(t) \rangle = N(t=0) \cdot 2^{-\frac{t}{t_{1/2}}} \Rightarrow \frac{\langle A(t) \rangle}{A(0)} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\tau\left({}^{238}_{92}U\right) = 4.5 \cdot 10^9\, a$$

Instant Quiz: Answers

Initial number of unstable (radioactive) nuclei in a sample = $N(t = 0)$

Any one of them may, or may not, have decayed by time $t > 0$

$\rightarrow$ Can make only probabilistic *predictions about average # remaining nuclei*

$$\langle N(t) \rangle = N(0) \cdot e^{-\lambda \cdot t}$$

Questions:

a) What is the probability for a given nucleus to decay at a given time t ?

b) What is the fraction $f_d(t)$ for the sample nuclei that have decayed and the fraction, $f_s(t)$, that have avoided ("survive") decay at time $t>0$?

c) Calculate the probabilistic mean lifetime τ of individual nuclei in the sample.

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Questions:

a) What is the probability for a given nucleus to decay at a given time t ?

$$\textit{identical nuclei} \rightarrow \frac{dP_d(t)}{dt} = \frac{A(t)}{N(t)} = \frac{\lambda \cdot N(t)}{N(t)} = \lambda \quad \rightarrow \quad P_d(t) = \lambda \cdot t \quad ?$$

b) What is the fraction $f_d(t)$ for the sample nuclei that have decayed and the fraction, $f_s(t)$, that have avoided ("survive") decay at time $t>0$?

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b) What is the fraction $f_d(t)$ for the sample nuclei that have decayed and the fraction, $f_s(t)$, that have avoided ("survive") decay at time $t>0$?

$$f_s(t) = \langle N(t) \rangle / N(0) = e^{-\lambda \cdot t} \rightarrow f_d(t) = 1 - \langle N(t) \rangle / N(0) = 1 - e^{-\lambda \cdot t}$$

c) Calculate the probabilistic mean lifetime τ of individual nuclei in the sample.

Instant Quiz: Answers

Initial number of unstable (radioactive) nuclei in a sample = *N(t = 0)*

Any one of them may, or may not, have decayed by time *t > 0*

→ Can make only probabilistic *predictions about average # remaining nuclei*

$$\langle N(t)\rangle = N(0)\cdot e^{-\lambda\cdot t}$$

Questions:

a) What is the probability for a given nucleus to decay at a given time *t* ?

$$\textit{identical nuclei} \rightarrow \frac{dP_d(t)}{dt} = \frac{A(t)}{N(t)} = \frac{\lambda\cdot N(t)}{N(t)} = \lambda \qquad \rightarrow \quad P_d(t) = \lambda\cdot t$$

b) What is the fraction $f_d(t)$ for the sample nuclei to have decayed and the fraction, $f_s(t)$, to have avoided ("survive") decay at time *t>0*?

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c) Calculate the probabilistic mean lifetime τ of individual nuclei in the sample.

$$\tau := \langle t\rangle_t = \int_0^\infty t\cdot N(t)dt \Big/ \int_0^\infty N(t)dt = \int_0^\infty t\cdot e^{-\lambda\cdot t}dt \Big/ \int_0^\infty e^{-\lambda\cdot t}dt = \lambda^{-1}$$

Quiz: Radioactivity of Pu-239

Consider pure sample of *0.5*g of Pu-239 (mono-isotopic).
Decay constant $\lambda = 9.1 \cdot 10^{-13}\ s^{-1}$ Unit $1\ s^{-1} = 1Bq$ (Bequerel)

Calculate activity A_0 at production *(t≈0)* and at 100years later.

1. Derive *N(t=0)* from weight =0.5g.

$$N(0) = \frac{0.5g}{239\,g/mol} = 2.09 \cdot 10^{-3} mol = 2.09 \cdot 10^{-3} mol \cdot \frac{6.022 \cdot 10^{23}}{mol} = 1.26 \cdot 10^{21}\ nuclei$$

2. Calculate activity *A(t=0)* of sample

$$A(0) = \lambda \cdot N(0) = 9.1 \cdot 10^{-13} s^{-1} \cdot 1.26 \cdot 10^{21} = 1.15 \cdot 10^{9} s^{-1} = 1.15 \cdot 10^{9} Bq$$

3. Calculate activity *A(t)* of same sample, but at *t=100a*

$$A(100a) = A(0) \cdot \exp\left\{-9.1 \cdot 10^{-13} s^{-1} \cdot 100a\right\}$$

$$100a = 100a \cdot 3.154 \cdot 10^{7}\ s/a = 3.154 \cdot 10^{9} s$$

$$\lambda t = 2.87 \cdot 10^{-3} \rightarrow \exp\left\{-2.87 \cdot 10^{-3}\right\} = 0.997 \rightarrow A(100a) = 0.997 \cdot A(0)$$

Exercise: Determine Actual Activity

Production date
December 2009

Unit of radioactivity $1Curie = 1Ci = 3.7.10^{10}\ decays/s$

Determine the actual activity of this Na-22 source today. Include uncertainty.

Production date
August 2013

Determine the actual activity of this Co-60 source today. Include uncertainty.

Exercise: Determine Actual Activity

Production date
December 2009

Unit of radioactivity $1Curie = 1Ci = 3.7.10^{10}\ decays/s$

Determine the actual activity of this Na-22 source today. Include uncertainty.

$$Halflife\ t_{1/2} = 2.6a = 2.6 \cdot 365.24d = 9.50 \cdot 10^{2} d$$

$$t = 12 \cdot 365.24d + 20d = 4.40 \cdot 10^{3} d$$

$$t/t_{1/2} = 4.64 \rightarrow A(t) = A(0) \cdot 0.5^{4.64} = 4.02 \cdot 10^{-2} \mu Ci$$

$$Uncertainty\ \#\, days\ in\ Dec/2 = \pm 15.5d \triangleq \pm 0.3\%$$

Production date
August 2013

Determine the actual activity of this Co-60 source today. Include uncertainty.

Exercise: Determine Actual Activity

Production date
December 2009

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$$Uncertainty\ \#days\ in\ Dec/2 = \pm 15.5d \triangleq \pm 0.3\%$$

Production date
August 2013

Determine the actual activity of this Co-60 source today. Include uncertainty.

$$Halflife\ t_{1/2} = 5.27a = 5.27 \cdot 365.24d = 1.925 \cdot 10^3 d$$

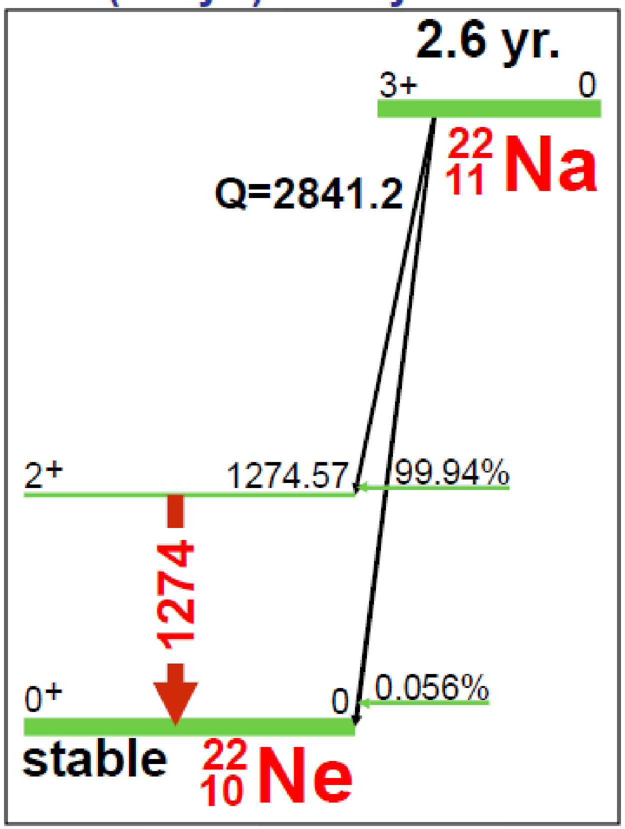
$$t = 8 \cdot 365.24d + 144d = 3.06 \cdot 10^3 d$$

$$t/t_{1/2} = 1.59 \rightarrow A(t) = A(0) \cdot 0.5^{1.59} = 0.33 \mu Ci$$

$$Uncertainty\ \#days\ in\ Aug/2 = \pm 15d \triangleq \pm 0.5\%$$

Decay Scheme Na-22

^{22}Na(2.6 yr.) Decay Scheme



GAMMA-RAY ENERGIES AND INTENSITIES

Nuclide: ^{22}Na — Half Life: 2.6019(4) yr.

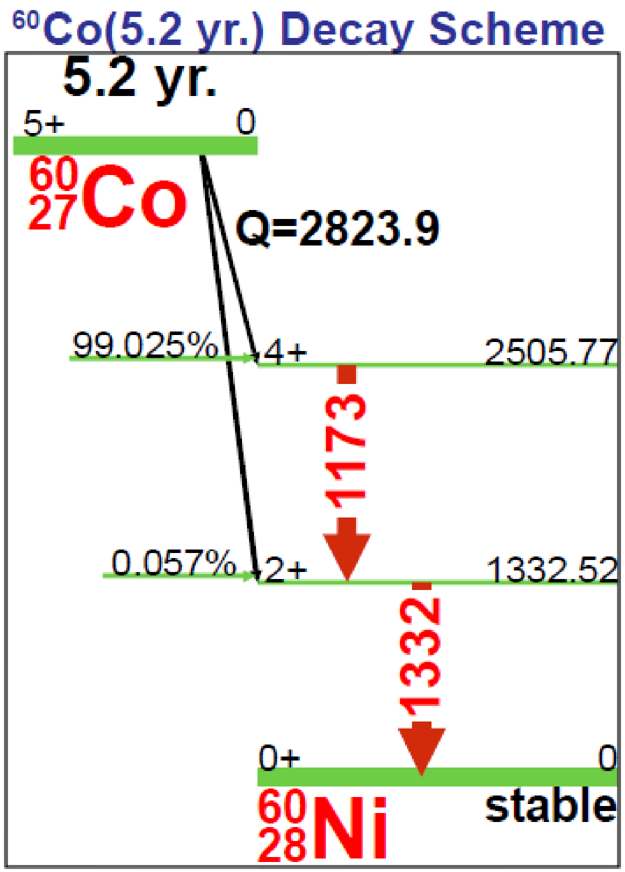
Detector: 55 cm^3 coaxial Ge (Li) — Method of Production: Ne(^{3}He,p)

	E_γ (keV)	$\sigma\ E_\gamma$	I_γ (rel)	I_γ (%)	$\sigma\ I_\gamma$	S
Ann.	511.006		100	178.0	0.6	1
	1274.53	0.02	62.2	99.944	0.014	1

E_γ, σE_γ, I_γ, σI_γ - 1998 ENSDF Data

https://gammaray.inl.gov/SiteAssets/catalogs/ge/pdf/na22.pdf

Decay Scheme Co-60



GAMMA-RAY ENERGIES AND INTENSITIES

Nuclide: ^{60}Co | Half Life: 5.2714(5) yr.

Detector: 55 cm^3 coaxial Ge (Li) | Method of Production: ^{59}Co(n,γ)

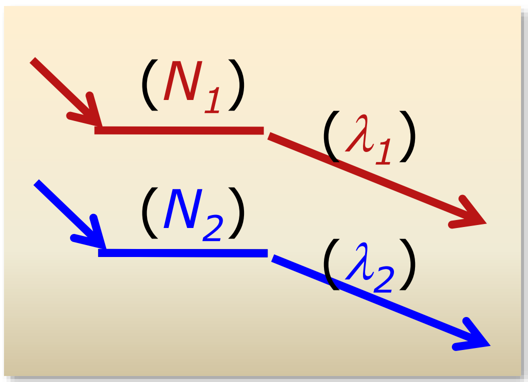
E_γ (keV)	σ E_γ	I_γ (rel)	I_γ (%)	σ I_γ	S
346.93	0.07		0.0076	0.0005	4
826.28	0.09		0.0076	0.0008	4
1173.237	0.004	100	99.9736	0.0007	1
1332.501	0.005	100	99.9856	0.0004	1
2158.77	0.09		0.0011	0.0002	4
2505.					4

E_γ, σE_γ, I_γ, σI_γ - 1998 ENSDF Data

https://gammaray.inl.gov/SiteAssets/catalogs/ge/pdf/co60.pdf

Sum Radioactivity

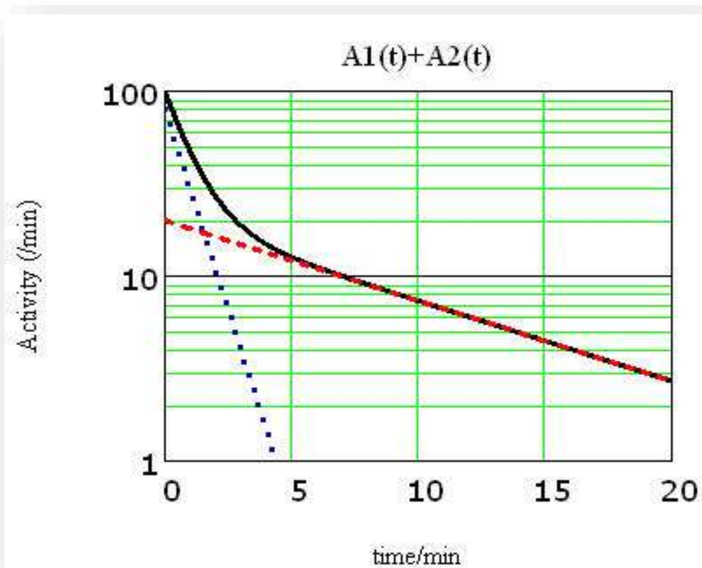
Genetically independent species:
Sample with 2 components (N_1, N_2) → same type of radiation (γ-rays)



$$A_i(t) = A_i(0) \cdot e^{-\lambda_i \cdot t} \quad (i = 1, 2)$$

Total activity :

$$A(t) = A_1(0) \cdot e^{-\lambda_1 \cdot t} + A_2(0) \cdot e^{-\lambda_2 \cdot t}$$

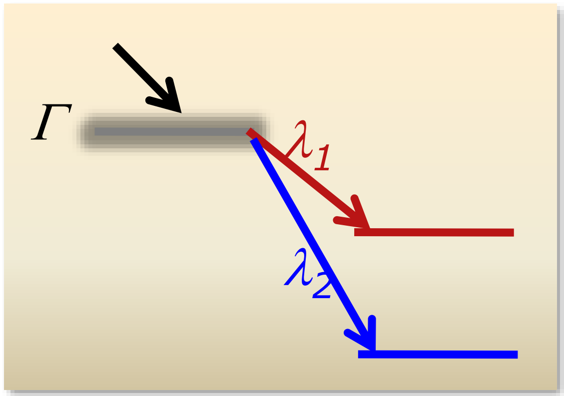


Decompose total decay curve → λ_1, λ_2.

Simultaneous fit or deduce constant λ_2 for "shallow" decay first

Branching Decay

Genetically dependent species: Sample depopulated by 2 decay paths (λ_1, λ_2)



$$\lambda = \lambda_1 + \lambda_2$$

$$\Gamma = \Gamma_1 + \Gamma_2 \quad \text{"level width"}$$

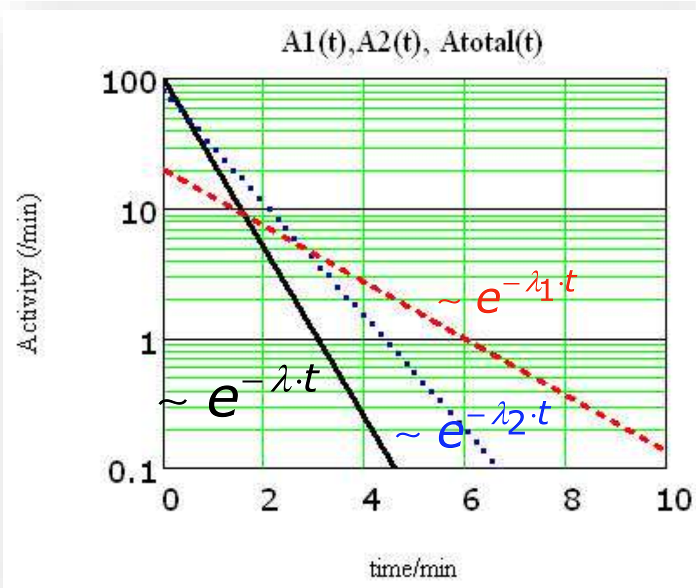
$$\frac{dN(t)}{dt} = -\lambda \cdot N(t) = -(\lambda_1 + \lambda_2) \cdot N(t)$$

$$N(t) = N(0) \cdot e^{-\lambda \cdot t} = N(0) \cdot e^{-(\lambda_1+\lambda_2) \cdot t}$$

$$A(t) = \lambda \cdot N(t) = \lambda \cdot N(0) \cdot e^{-\lambda \cdot t} = A_1(t) + A_2(t)$$

Partial activities :

$$\rightarrow A_i(t) = \lambda_i \cdot N(0) \cdot e^{-\lambda \cdot t} \qquad (i = 1, 2)$$

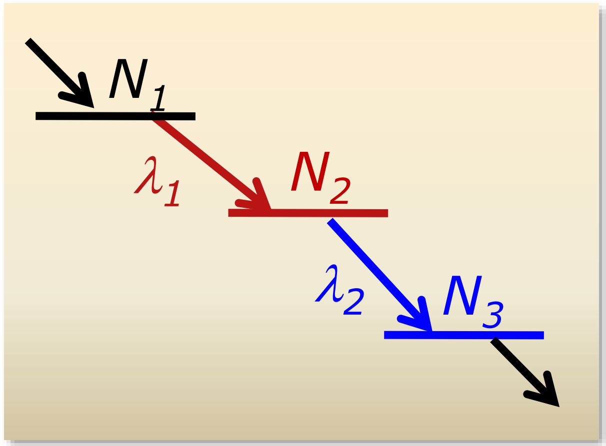


Partial decay rates/half lives:

$$\frac{A_i(t)}{A(t)} = \frac{\lambda_i \cdot N(t)}{\lambda \cdot N(t)} = \frac{\lambda_i}{\lambda} \qquad \left(t_{1/2}\right)_i = \frac{0.693}{\lambda_i}$$

Identify radiation type *i* to measure partial decay rates/half lives.

Genetically Related Decay Chain



$$\frac{dN_i(t)}{dt} = \lambda_{i-1}N_{i-1}(t) - \lambda_i N_i(t)$$

Gain and loss for *i-th* daughter

Coupled DEq. For populations N_i of nuclei in chain

$$N_1(t) = c_{11}\cdot e^{-\lambda_1 \cdot t} \qquad P(parent)$$

$$N_2(t) = c_{21}\cdot e^{-\lambda_1 \cdot t} + c_{22}\cdot e^{-\lambda_2 \cdot t} \qquad P(1.daughter)$$

$$\vdots$$

$$N_k(t) = \sum_{m=1}^{k} c_{km}\cdot e^{-\lambda_m \cdot t} \qquad P((k-1).daughter)$$

$$k+1: \text{final grand daughter (stable)}$$

Boundary condition

$$N_i(0) = c_{i1} + c_{i2} + \ldots + c_{ii}$$

$$\rightarrow \text{determines } c_{ii}$$

Recursion Relations

$$c_{ij} = c_{i-1,j}\cdot\frac{\lambda_{i-1}}{\lambda_{i-1}-\lambda_j}$$

$$k = 2: \quad N_1(t) = N_1(0)\cdot e^{-\lambda_1 \cdot t}$$

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2 - \lambda_1}\cdot\left(e^{-\lambda_1 \cdot t} - e^{-\lambda_2 \cdot t}\right)$$

Check by differentiation

Activities and Equilibrium in Decay Chains

$$k = 2: \quad N_1(t) = N_1(0)\cdot e^{-\lambda_1 \cdot t}$$

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2-\lambda_1}\cdot\left(e^{-\lambda_1\cdot t} - e^{-\lambda_2\cdot t}\right)$$

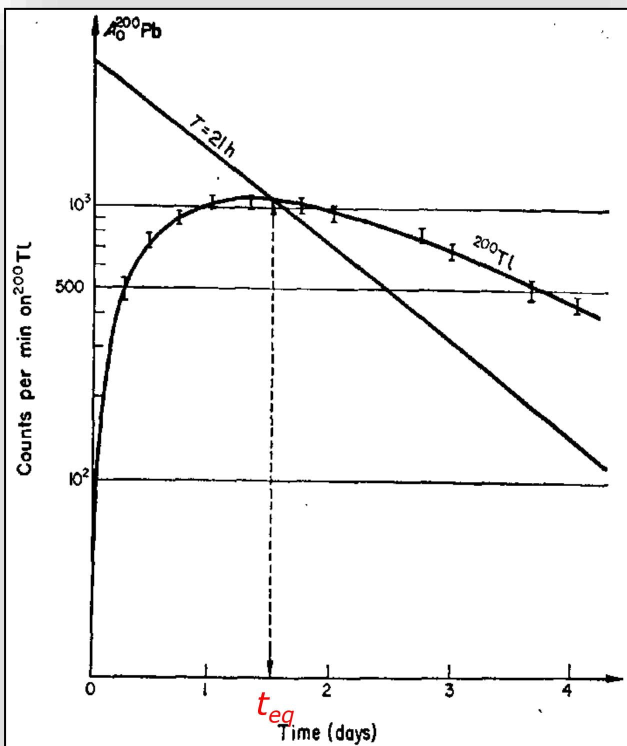
$$N_3(t) = N_1(0)\left\{1 + \frac{\lambda_1}{\lambda_2-\lambda_1}e^{-\lambda_2\cdot t} - \frac{\lambda_2}{\lambda_2-\lambda_1}e^{-\lambda_1\cdot t}\right\}$$

$$A_1(t) = \lambda_1 N_1(t) = A_1(0)\cdot e^{-\lambda_1\cdot t} = -\frac{dN_1}{dt}$$

$$A_2(t) = \lambda_2 N_2(t) = A_1(0)\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\left(e^{-\lambda_1\cdot t} - e^{-\lambda_2\cdot t}\right)$$

$$A_2(t) \neq -\frac{dN_2}{dt} \qquad | \; A_3(t) = 0 \quad (\lambda_3 = \infty)$$

$$\frac{A_2(t)}{A_1(t)} = \frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\left(1 - e^{-(\lambda_2-\lambda_1)\cdot t}\right) \xrightarrow[t\to\infty]{} \left(\frac{\lambda_2}{\lambda_2-\lambda_1}\right)$$



Transitory/secular Equilibrium

$$A_1(t_{eq}) = A_2(t_{eq}) \quad \rightarrow \quad t_{eq} = \frac{\ell n(\lambda_1/\lambda_2)}{(\lambda_1-\lambda_2)}$$

^{200}Pb: $t_{1/2}$=21h → ^{200}Tl: $t_{1/2}$=26h → ^{200}Hg

$$\lambda_1 = 0.693/21h = 9.17\cdot 10^{-6}\, s^{-1} > \lambda_2$$

$$\lambda_2 = 0.693/26.4h = 7.29\cdot 10^{-6}\, s^{-1}$$

$$t_{eq} = \frac{0.229}{1.88\cdot 10^{-6} s^{-1}} = 1.22\cdot 10^{5} s = 1.41d$$