Algebraic properties of Artin-Mazur zeta function in positive characteristic

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Description

In the previous decades, a lot of effort was put into studying the properties of the Artin-Mazur zeta function (AMZF). For example, [Hin94] showed that rational maps of degree at least 2 acting on a Riemann sphere have rational AMZF and that its formula depends only on the number of parabolic cycles. More recently, [KH07] and [Ber21] showed that, in a measure-theoretic sense, almost every diffeomorphism has a divergent AMZF.

However, little is known about the AMZF in positive characteristic and the current progress is based on extremely specific cases. [Bri16] showed that the AMZF for Chebyshev polynomials, power maps, and Lattès maps is transcendental over $\mathbb{Q}(t)$, and separable additive map f has rational AMZF when $f'(0)$ is transcendental over $\mathbb{F}_p$. In his proof, Bridy relied on Christol's [Chr79] and Cobham's [Cob69] theorems, which relate algebraic field theory and automata theory. Inspired by Bridy's work, in this Honors Thesis we will investigate the algebraic properties of AMZF for map $f = x^2 + 1$ over $\mathbb{Q}(t)$ for fields of characteristic $p \geq 3$.

In our work, we will continue applying techniques from automata theory introduced by Bridy, however, we will also use some other concepts from automata theory described in [AS03]. In addition to this, we will introduce new ideas in order to get better understanding of number of periodic points of period n for the map $f = x^2 + 1$. Namely, we will apply the concept of the generalized Sen's theorem proved by [LR16] and the idea of bounding the lower ramification numbers using techniques from [NR20] and [Nor21]. Combining these techniques, we are hoping to get enough information about the number of periodic points of period n for $f = x^2 + 1$ to effectively apply machinery from automata theory and get new insights about the algebraic properties of the AMZF for f.

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