

## Stirling's Formula

Stirling's formula is an approximation to the factorial  $n!$ , which is difficult to evaluate exactly for very large integers  $n$ . Neglecting higher-order terms, this formula predicts

$$\ln n! \approx \frac{1}{2} \cdot \ln(2\pi) + \left(n + \frac{1}{2}\right) \cdot \ln n - n \quad (1)$$

This is not the only formula used to approximate  $f(n) := n!$ . The one in Eq. 2 stitches the exact formula at  $n=22$  to an approximate expression employing the Heaviside function  $\Phi$

$$f(n) := \Phi(22 - n) \cdot n! + \Phi(n - 22) \cdot \left[ \left( \frac{n}{2.71828} \right)^n \cdot \sqrt{2 \cdot \pi \cdot n} \cdot \left( 1 + \frac{1}{12 \cdot n} + \frac{1}{288 \cdot n \cdot n} \right) \right] \quad (2)$$

The graph compares the exact factorial (solid line) to Eq.1 (circles) and Eq.2 (x) on a semi- $\text{Log}_{10}$  scale. Obviously, Eq. 1 suffices in many cases.

