2 March 2023

9:40-10:55

Physical Chemistry II (Chm 252/442)

Midterm Exam 1

(***Chm252****: Answer all questions, except those marked "AC Chm442".*
Chm442 *augmented credit: Answer all questions.*
Show all work explicitly on the sheets provided, front & back.)

Respecting the University of Rochester policies on Academic Honesty, I affirm that this is submitted as my own work, and that I have neither received assistance by another person nor consulted sources other than my own prepared summary sheet, the allowed course notes and text books.

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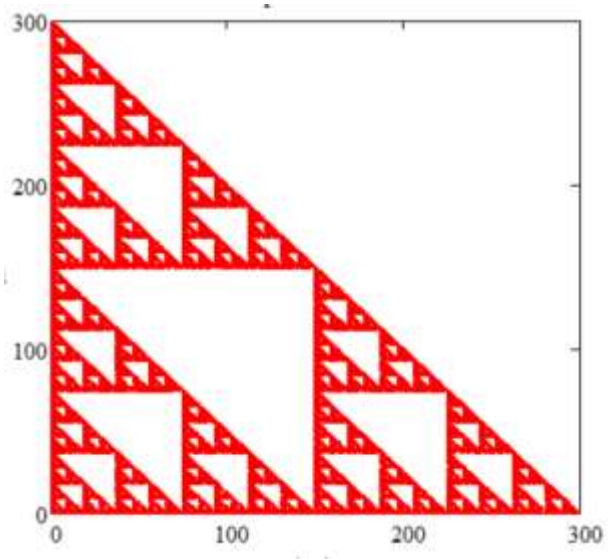
Signed (Student Name and ID)

1. Fractal Structures



Often, hollow structures have mechanical stability superior to solid material, at lower weight and material cost. Consider the planar structured shape in the figure, which is plotted to scale (300u **X** 300u). The intricate hole structure is self-similar (fractal) on decreasing length scales.

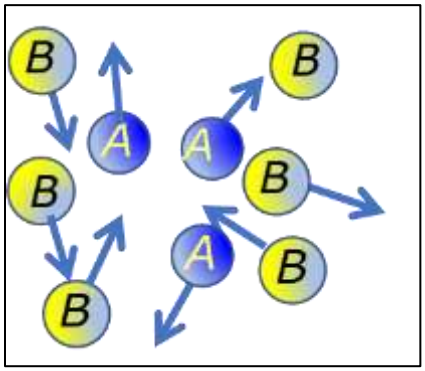
a) Briefly describe the process by which the structure differentiates itself, from one scale to the next smaller scale, starting from a solid structure on the largest (300u **X** 300u) scale,

b) How many similar solid shapes are visible on the plot at the given magnification?

c) Determine the dimensional factor ***s*** governing the transformation from scale to scale.

d) (**AC Chm 442**) Determine the fractal dimension ***d*** and the density ρ of the structure shown in the figure.

2. Autocatalytic Reactions



Consider the autocatalytic reaction $A + B \underset{k_r}{\overset{k_f}{\rightleftarrows}} B + B$, with time ($t$) dependent concentrations $C_i(t)$; i = *A, B*, and reaction rate constants $k_f > k_r$. As shown in class, with a conveniently rescaled concentration, $C_B \to C$, and constant, k, the reaction progress can be described in terms of the rate equation

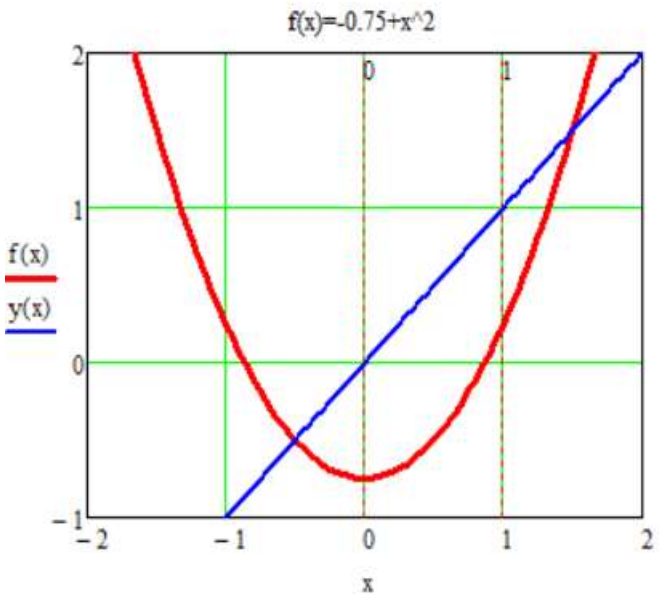
$$dC/dt = k \cdot C \cdot (1 - C)$$

a) Determine the concentrations C at all stationary states of the reactive system.

b) Explain which of these states are stable against small fluctuations in C and why.

c) (**AC Chm 442**) Show equivalence of the above DEq for C(*t*) with $\frac{d}{dt} Ln\left[\frac{c(t)}{1-c(t)}\right] = 1.$

3. Analysis of a One-Dimensional Map



Consider the family of quadratic maps **Qc**: $x_{n+1} = c + x_n^2$ with *n=0,1,….* where ***c*** is a real constant. The maps are defined for the range $-2 \le x \le +2$. The figure shows a sample the map function ***f(x)***, together with ***y(x) = x***.

Tasks: Determine the map fixpoints for values of *c > 1/4, c = 1/4,* and *c < 1/4.* Briefly discuss their nature (attractor, repeller,…). Show 2 "cobweb" iteration trajectories.

a) The map constant has a value of ***c > 1/4***.

b) The map constant has a value of ***c = 1/4***

c) The map constant has a value of ***c < 1/4***

d) (**AC Chem442**) What phenomenon occurs for iterates on the map with ***c = -3/4*** ?

e) On the map shown above, use the graphical method discussed in class, draw two iterations (*n=0,1,…,5*), one with initial point $x_0 > 0$, and one with $x_0 < 0$.

Extra sheet for work.